prime factors, 500 per page, arranged in an obvious format. (One can see at once which $n^{2}+1$ are prime by the relative size of the corresponding listed factors.)

These factorizations relate to questions concerning reducible numbers, primes of the form $n^{2}+1$, formulas for $\pi$, and other questions surveyed in [1].

In [2] and [3] similar $p$-adic sieves were run for $n^{2} \pm 2, n^{2} \pm 3, n^{2}+4, n^{2} \pm 5$, $n^{2} \pm 6$, and $n^{2} \pm 7$ for $n=1(1) 180000$, but only statistical information was preserved, not the complete table of greatest prime factors.

## D. S.

1. DANIEL SHANKS, "A sieve method for factoring numbers of the form $n^{2}+1$," MTAC, v. 13, 1959, pp. 78-86.
2. DANIEL SHANKS, "On the conjecture of Hardy \& Littlewood concerning the number of primes of the form $n^{2}+a, "$ Math. Comp., v. 14, 1960, pp. 321-332.
3. DANIEL SHANKS, "Supplementary data and remarks concerning a Hardy-Littlewood conjecture," Math. Comp., v. 17, 1963, pp. 188-193.

13 [9].-J. D. Swift, Table of Carmichael Numbers to $10^{9}$, University of California at Los Angeles, ms. of 20 pages, $8 \frac{1}{2}^{\prime \prime} \times 11^{\prime \prime}$, deposited in the UMT file.

A Carmichael number, CN , is a composite number $n$ such that $a^{n-1} \equiv 1$ $(\bmod n)$ whenever $(a, n)=1$. Carmichael numbers are starred in Poulet's table [1] of pseudoprimes less than $10^{8}$. The present table corrects that table and extends the range to $10^{9}$. The CN's are given with their prime factors.

Calculations were performed on a CDC 1604 made available by IDA, in Princeton. The computer programs used depended explicitly on congruence properties of CN's with respect to their component primes rather than on the pseudoprimality with respect to any particular base. A different routine was run for each possible number of primes occurring in the factorization, from 3 (the absolute minimum) to 6 (the effective maximum defined by the upper limit of the table).

For example, consider $n=p_{1} p_{2} p_{3}=\left(r_{1}+1\right)\left(r_{2}+1\right)\left(r_{3}+1\right)$. The basic criteria are that $r_{i} \mid p_{j} p_{k}-1$ where $i, j, k$ is a permutation of $1,2,3$. For a fixed choice of $p_{1}$ (assuming $p_{1}<p_{2}<p_{3}$ ), bounds on the limits of the calculation are obtained. In this simplest case an explicit bound is available:

$$
p_{1} p_{2} p_{3} \leqslant\left(p_{1}^{6}+2 p_{1}^{5}-p_{1}^{4}-p_{1}^{3}+2 p_{1}^{2}-p_{1}\right) / 2
$$

and this is actually a CN for $p_{1}=3,5,31, \cdots(?)$.
The total number of CN's less than or equal to each power of 10 is as follows:

| $x$ | $\mathrm{CN}(x)$ | ratio |
| :---: | :---: | :---: |
| $10^{4}$ | 7 |  |
| $10^{5}$ | 16 | 2.3 |
| $10^{6}$ | 43 | 2.7 |
| $10^{7}$ | 105 | 2.4 |
| $10^{8}$ | 255 | 2.4 |
| $10^{9}$ | 646 | 2.5 |

The known results thus appear to suggest an asymptotic relation for $\mathrm{CN}(x)$ as of the order of $C x^{0.4}$, which is much smaller than has been conjectured by Erdös [2]. In this connection, the change from 2.43 to 2.53 in the ratios of the last orders of magnitude computed may be significant.

## AUTHOR'S SUMMARY

1. P. POULET, "Table des nombres composés vérifiant le théorème de Fermat pour le module 2 jusqu'à $100.000 .000, " \operatorname{Sphinx}$, v. 8,1938 , pp. 42-52. For corrections see Math. Comp., v. 25, 1971, pp. 944-945, MTE 485; v. 26, 1972, p. 814, MTE 497.
2. P. ERDÖS, "On pseudoprimes and Carmichael numbers," Publ. Math. Debrecen, v. 4, 1956, pp. 201-206.

14 [9]-H. C. Williams \& C. R. Zarnke, A Table of Fundamental Units for Cubic
Fields, Scientific Report 63, University of Manitoba, Winnipeg, January 1973.
Table 1 gives the fundamental unit $\epsilon_{0}=\left(U+V \rho+W \rho^{2}\right) / T$ for all irreducible cubics $\rho^{3}=Q \rho+N$ having $|Q|, N \leqslant 50$ and a discriminant $D<0$. Table 3 gives $\epsilon_{0}$ for $\rho^{3}=A \rho^{2}+B \rho+C$ with $A,|B|,|C| \leqslant 10$ and $D<0$. For $D>0$ there are two fundamental units and Tables 2 and 4 give both of them for the same range of $Q, N$ and $A, B, C$, respectively.

These are the most extensive tables of cubic units known to me although for special types, such as cyclic or pure cubic fields, units have been computed that are not included here.

No attempt is made here to identify different $Q, N$ or $A, B, C$ that give the same field. That would be a valuable addition, especially if it gave the transformation taking one $\rho$ into another.
D. S .

15 [9].-Kenneth S. Williams \& Barry Lowe, Table of Solutions ( $x, u, v, w$ ) of the Diophantine System $16 p=x^{2}+50 u^{2}+50 v^{2}+125 w^{2}, x w=v^{2}-4 u v-u^{2}$, $x \equiv 1(\bmod 5)$ for Primes $p<10000, p \equiv 1(\bmod 5)$, Carleton University, Ottawa, 1974, manuscript of 13 pages deposited in the UMT file.

The authors tabulate the values $(x, u, v, w)$ of one of the four solutions of the Diophantine system in the title for all primes $p \equiv 1(\bmod 10)$ less than 10000 , the remaining three solutions being $(x,-u,-v, w),(x, v,-u,-w)$, and $(x,-v, u,-w)$. These solutions are obtained from the coefficients of the Jacobi function of order five which have been tabulated by Tanner [1] for $p<10000$. Two errors in Tanner's tables are noted and one in earlier tables.

A derivation of the well-known linear relationship between these coefficients (which are in fact Jacobsthal sums) and the solutions $x, u, v, w$ is also given.

It should be pointed out that Joseph Muskat has obtained the solutions $(x, u, v, w)$ a number of years ago for all $p \equiv 1(\bmod 10)$ for $p<50000$ from

